B.Sc Physics (Hons) Part I
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Q (a) Differentiate between Simple Harmonic Motion and oscillatory motion.
(b) Define general differential equation of motion of a simple harmonic oscillator and its various solution.

Ans (a) Oscillatory motion:- A motion which repeats itself after regular interval of time is called periodic motion. If a body in periodic motion excutes to and fro motion about a fixed reference point, it is said an oscillatory motion.

The term oscillatory motion is not ~~restric~~ restricted only to displacement of a mechanical oscillator, but it may be any physical quantity. For example in electrical system an oscillatory variation of charge, current or voltage may be takes place.

SIMPLE HARMONIC MOTION :-

When a body moves such that its acceleration is always directed towards a certain fixed point and varies directly as its distance from that point, the body is said to be excute S.H.M.

For such a motion to take place the force acting on the body should be directed towards the fixed point and should also be proportional to the displacement. The function of the force is to bring the body back to its equilibrium position and hence this force is often known as restoring force.

(b) Differential equation of Simple Harmonic Oscillator:-

Let a particle having mass 'm' executing S.H.M. If y be the displacement of the particle from equilibrium position at any instant 't', The restoring force F, $F \propto y \Rightarrow F = -Ky$

where K = force constant or spring constant. (−ve) sign is used to indicate that the direction of the force is opposite to direction of increasing displacement.

Force constant K is defined as the restoring force per unit displacement

$$K = \frac{F}{y} \qquad \because F = -Ky$$

$$m\frac{d^2y}{dt^2} = -Ky \Rightarrow \frac{d^2y}{dt^2} + \frac{K}{m}y = 0$$

let $\frac{K}{m} = \omega^2$

$$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{------} \quad (I)$$

This is the differential equation of motion of a simple harmonic oscillator

To find the solution of differential eq^n

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

$$2\frac{dy}{dt}\frac{d^2y}{dt^2} + \omega^2 2y\frac{dy}{dt} = 0$$

on integrating, we have

$$\left(\frac{dy}{dt}\right)^2 = -\omega^2 y^2 + c \quad \text{------} \quad (II)$$

c = constant of integeration

when the displacement is maximum at $y = a$. Here a = amplitude of the oscillating particle, if $\frac{dy}{dt} = 0$

from (II) $\left(\frac{dy}{dt}\right)^2 = \omega^2(a^2 - y^2)$. $y = a$, and $\frac{dy}{dt} = 0$

$$\therefore c = a^2\omega^2$$

Again from (II) $\left(\frac{dy}{dt}\right)^2 = \omega^2(a^2 - y^2)$

$$\therefore \frac{dy}{dt} = \omega\sqrt{a^2 - y^2} \quad \text{------} \quad (III)$$

This eq^n gives the velocity of the particle executes S.H.M.

$$\therefore \frac{dy}{\sqrt{a^2 - y^2}} = \omega dt$$

on integrating

$$\sin^{-1}\frac{y}{a} = \omega t + \phi$$

$$\therefore y = a\sin(\omega t + \phi) \quad \text{------} \quad (IV)$$

The term $(\omega t+\phi)$ represents the total phase of the particle at time 't' and ϕ is known as the initial phase or phase constant. If the time is recorded from the instant when $y=0$ and increasing then $\phi=0$

Other Solution :→

The equation $y = a\sin(\omega t+\phi)$ is just one sol^n of the differential eq^n $\frac{d^2y}{dt^2} + \omega^2 y^2 = 0$

An equally valid sol^n of this equation is

$y = a\cos(\omega t+\phi) \Rightarrow y = a\sin(\omega t + \phi)$ --- (V)

$y = a(\cos\phi \cdot \sin\omega t + \sin\sin\omega t \cdot \cos\phi)$

$y = A\sin\omega t + B\cos\omega t$ --- (VI)

which is another valid solution, in which

$A = a\cos\phi$ and $B = a\sin\phi$

$\therefore a = \sqrt{A^2+B^2}$ and $\phi = \tan^{-1}\left(\frac{B}{A}\right)$

Similarly, by expanding $y =$ ~~$a\sin(\omega t+\phi)$~~ $a\cos(\omega t+\phi)$, we get

$y = a(\cos\phi\cos\omega t - \sin\phi\cdot\sin\omega t)$

$y = A\sin\omega t + B\cos\omega t$

where $A = -a\sin\phi$ and $B = a\cos\phi$

$\therefore a = \sqrt{A^2+B^2}$ & $\phi = \tan^{-1}(-A/B)$

The various general sol^n of diffrential eq^n

(1) The Sin - Cosine form are

$y = a\sin(\omega t+\phi) \Rightarrow y = a\cos(\omega t+\phi)$

$y = A\sin\omega t + B\cos\omega t$

(11) Exponential form :-

we can put diffrential eq^n (1) in the operator form by substituting $\frac{d}{dt} = D$

or, $\frac{d^2}{dt^2} = D^2$ we get $D^2y + \omega^2 y = 0$ $\therefore D^2 = -\omega^2$ $\therefore D = \pm i\omega$

Hence in general sol^n of eq^n (1) becomes

$y = Ae^{i\omega t} + Be^{-i\omega t}$

In order to that this solution may give a real value of y, A and B must be complex conjugates of each other i.e

$A = a + ib$ and $B = a - ib$

A second form of the general sol^n of eq^n (1) is

$y = ae^{i(\omega t+\phi)}$

of two constants a and ϕ ~~st~~ satisfies eq^n (1)

The sol^n of diffrential eq^n (1) in exponential form are

$y = Ae^{i\omega t} + Be^{-i\omega t}$

& $y = ae^{i(\omega t+\phi)}$